

Application of Multi-Criteria Decision Analysis and Inoperability Input-Output Models for Disaster Risk Management

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Science for Climate Resilient Communities

BACKGROUND

The growing threat of natural hazards, resulting from climate change, and the disasters they may bring have become a matter of national and international concern particularly for countries located along the Pacific Ring of Fire (Asian Development Bank, 2013).



BACKGROUND



- 5th in the global climate risk index
- 3rd most high-risk Asian developing country (disaster-driven economic loss)



BACKGROUND

- According to Kelman, Gaillard, and Mercer (2015), disaster risk is largely determined by vulnerability, which is primarily driven by factors such as poverty.
- Conversely, disasters have also been identified as a major cause of poverty since a majority of the poor depend on agriculture as a means of living.
- Poverty incidence and economic losses are worsened because economic sectors are interdependent thus causing disruptions in one sector to cascade to previously unaffected sectors.



BACKGROUND

Disaster risk analysis is imperative to ensure that limited resources are allocated efficiently.

INPUT-OUTPUT ANALYSIS

- To capture the sector-specific effects of natural hazards
 - Great Hanshin Earthquake of Kobe, Japan in 1995 (Okuyama et al, 2004)
 - Great East Japan Earthquake (Kajitani and Tatano, 2014)
 - Hurricane Katrina of the United States of America (Crowther, et al., 2007)



BACKGROUND

INPUT-OUTPUT ANALYSIS

- contains a key characteristic necessary for disaster-risk estimation and this is its ability to fully capture the relationship and interactions of economic sectors
- widely used tool for disaster impact and risk evaluation



SIGNIFICANCE

Existing web-based open-source tools, which were created to help policymakers make more-equipped disaster risk management decisions, do not make use of this model.



SIGNIFICANCE

Since existing tools only provide a general scope of the impact of disasters, there is still a need to develop a science-based tool that is able to estimate the economic impact of disasters caused by natural hazards to various sectors.



OBJECTIVES

Disaster-Risk Estimation and Analysis with Leontief Models (Disaster-REALM)

- first web-based freeware for disaster risk assessment using
- fully captures the interdependence of each sector in the
- determines a disaster's sector-specific effects:
 - Economic Loss
 - Degree of Inoperability



OBJECTIVES

Disaster-Risk Estimation and Analysis with Leontief Models (Disaster-REALM)

- Users can simulate disasters by simply specifying the perturbation
- Natural Hazards: typhoons, earthquakes, droughts, and
- Other Scenarios: workforce absenteeism, pandemic outbreaks, production



OBJECTIVES

Disaster-Risk Estimation and Analysis with Leontief Models (Disaster-REALM)

- User-friendly and easily accessible
- Immediate scientific estimates of a disaster's sector-specific
- Aids stakeholders in determining which sectors to prioritize costs of each policy
- Accessible through **disaster-realm.net**



METHODOLOGIES

Disaster-Risk Estimation and Analysis with Leontief Models (Disaster-REALM)

- Leontief's 1973 Nobel Price-winning Input-Output (I-O) Model
- Two components:
 - Inoperability Input-Output Modelling Tool (static)
 - Dynamic Inoperability Input-Output Modelling Tool (dynamic)



Basic Leontief Input-Output Model

- Leontief (1936) models the flow of transactions among buyers and sellers by assuming that each sector produces homogeneous goods and that each firm produces a unit of output from a fixed amount of inputs (Miller and Blair, 2009).



Basic Leontief Input-Output Model

The model is expressed as: $\mathbf{x} = \mathbf{Ax} + \mathbf{y}$

where \mathbf{A} = technical coefficients matrix (how much input sector j needs from sector i to produce its output)

\mathbf{x} = total output vector (total output produced by sector i)

\mathbf{y} = final demand vector (final demand for sector i 's output)



Basic Leontief Input-Output Model

$$\begin{array}{l}
 \mathbf{x} = \mathbf{A} \mathbf{x} + \mathbf{y} \\
 \begin{array}{l} \text{Agriculture} \\ \text{Industry} \\ \text{Service} \end{array} \begin{bmatrix} 10 \\ 18 \\ 7 \end{bmatrix} = \begin{bmatrix} 0.11 & 0.14 & 0.02 \\ 0.06 & 0.30 & 0.14 \\ 0.12 & 0.18 & 0.31 \end{bmatrix} \begin{bmatrix} 23 \\ 82 \\ 111 \end{bmatrix} + \begin{bmatrix} 7 \\ 40 \\ 59 \end{bmatrix}
 \end{array}$$

Sector i's output
of inputs sector i supplies to the other economic sectors
of output it produces for final consumption (HH, Government, Exports)



Inoperability Input-Output Model

Developed by Haimes and Jiang (2001)

Inoperability is defined as:

- “the inability of a system to perform its intended function” as a result of a perturbation or external shock (Haimes and Jiang, 2001)
- the normalized production loss caused by perturbations
- failure to produce output



Inoperability Input-Output Model

Inoperability is defined as: $\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^*$

where \mathbf{q} = inoperability vector (ranges from 0 (normal state of a system) to 1 (total failure))

\mathbf{A}^* = interdependency matrix (represents the additional inoperability sector i contributes to sector j)

\mathbf{c}^* = perturbation vector (reduction in demand for each sector's output because of shocks like disasters)



Inoperability Input-Output Model

Inoperability is defined as:

$$\underbrace{\mathbf{q}}_{\text{Sector i's failure to produce}} = \underbrace{\mathbf{A}^* \mathbf{q}}_{\text{Amount of indirect failure it experiences (from the sectors it depends on)}} + \underbrace{\mathbf{c}^*}_{\text{Amount of direct failure (Initial reduction in the final demand for its output)}}$$



Inoperability Input-Output Model

Economic Loss: monetary equivalent of the inoperability caused by a perturbation

$$\begin{aligned} \text{EL} &= \mathbf{q} \mathbf{x} \\ &= \text{inoperability} * \text{sector } i\text{'s normal output level} \end{aligned}$$



Dynamic Inoperability Input-Output Model

- developed by Lian and Haimes (2006)
- accounts for the dynamic nature of the effects caused by a perturbation
- accounts for an economy's slow recovery, as well as the additional disruptions that will trickle down from the time of the initial perturbation to the economy's full recovery at time T
- allows for a more realistic assessment of the risks caused by a disaster given the full duration and dynamic nature of the recovery process



Dynamic Inoperability Input-Output Model

The DIIM is defined by:

$$\mathbf{q}(t + 1) = \mathbf{q}(t) + \mathbf{K}[\mathbf{A}^* \mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t)]$$

Where \mathbf{q} = inoperability vector

\mathbf{A}^* = interdependency matrix

\mathbf{c}^* = perturbation vector

\mathbf{K} = resilience matrix

t = time indicator

Diagonal matrix of $k_i = \frac{\ln\left[\frac{q_i(0)}{q_i(T_i)}\right]}{T_i} * \frac{1}{(1-a_{ii})}$ or
the dynamic recovery process of sector i



Dynamic Inoperability Input-Output Model

The DIIM is defined by:

$$\underbrace{\mathbf{q}(t + 1)}_{\substack{\text{Sector i's} \\ \text{production} \\ \text{failure} \\ \text{at the next} \\ \text{time period}}} = \underbrace{\mathbf{q}(t)}_{\substack{\text{Sector i's} \\ \text{production} \\ \text{failure} \\ \text{at the current} \\ \text{time period}}} + \mathbf{K} \underbrace{\left[\mathbf{A}^* \mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t) \right]}_{\substack{\text{Sector i's recovery and} \\ \text{additional losses from the} \\ \text{current to the next time} \\ \text{period}}}$$



Dynamic Inoperability Input-Output Model

Economic Loss: Given that this model accounts for the dynamic recovery process of the economy, economic loss is the sum of the economic losses across **T** time periods. It is defined as:

$$EL_i = \sum_t [q_i(t) * x_i]$$





Disaster R.E.A.L.M.

Disaster Estimation and
Analysis with Leontief Models

ECONOMICS SECTOR

CASE STUDY:

5% reduction in
agricultural output
caused by a typhoon



[IIM Tool](#)
[DIIM Tool](#)

The damage brought by the disaster greatly affects one or more economic sector (inoperability)

Disaster hits the country

Due to the interdependence of the economic sector, other sectors are also affected (using the IIM)

Quantifies the effect of each sector due to the inoperability of one or more sector.

Rank each sector from the most affected sector to the least affected.

DISASTER RISK MANAGEMENT USING IIM



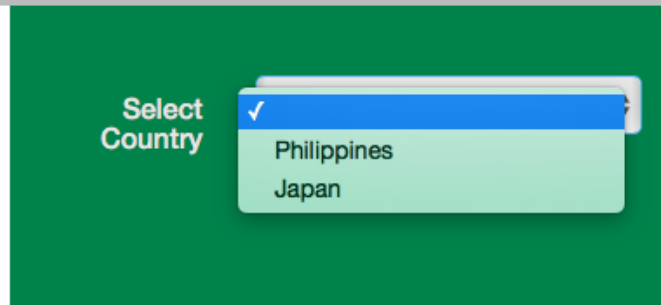
Inoperability Input-Output Model (IIM)

- The IIM Tool allows users to simulate an initial perturbation and measure the immediate impact of this perturbation to the economic system without accounting for the response or changes in the system over time.



Step 1: Select Country

IIM Tool



Step 2: Select Year and Disaggregation

IIM Tool

Select Country: Philippines

Select Year & Sector:

- 2000 Philippines 11 Sectors
- 2000 Philippines 60 Sectors
- 2000 Philippines 240 Sectors
- 2006 Philippines 11 Sectors
- 2000 Japan 13 Sectors
- 2011 Japan 13 Sectors
- 2011 Japan 53 Sectors
- 2011 Japan 80 Sectors



Step 3: Encode Initial Perturbation

IIM Tool

Select Country

Select Year & Sector

% Reduction in Total Output

Agriculture, Fishery and Forestry	<input type="text" value="5"/>
Mining and Quarrying	<input type="text" value="0"/>
Manufacturing	<input type="text" value="0"/>
Construction	<input type="text" value="0"/>
Electricity, Gas and Water	<input type="text" value="0"/>
Transportation, Communication and Storage	<input type="text" value="0"/>
Trade	<input type="text" value="0"/>
Finance	<input type="text" value="0"/>
Real Estate and Ownership of Dwellings	<input type="text" value="0"/>
Private Services	<input type="text" value="0"/>
Government Services	<input type="text" value="0"/>



IIM Tool

Select Country Philippines

Select Year & Sector 2000 Philippines 11 Sectors

% Reduction in Total Output

Agriculture, Fishery and Forestry

Mining and Quarrying

Manufacturing

Construction

Electricity, Gas and Water

Transportation, Communication and Storage

Trade

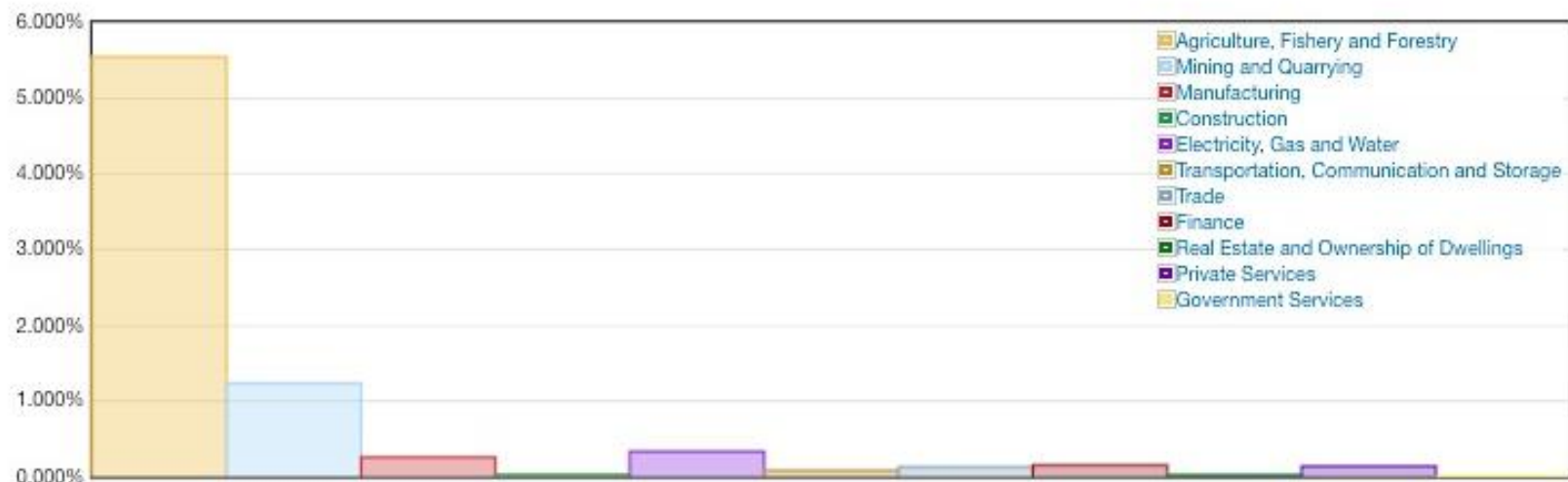
Finance

Real Estate and Ownership of Dwellings

Private Services

Government Services

Submit



[Export to CSV](#)

Sector	Final Inoperability Per Sector	Inoperability Rank	Economic Loss in Million Php	Economic Loss Rank
Agriculture, Fishery and Forestry	5.55%	1	₱3,809.71	1
Mining and Quarrying	1.23%	2	₱46.62	6
Manufacturing	0.25%	4	₱829.91	2
Construction	0.02%	9	₱6.75	9
Electricity, Gas and Water	0.33%	3	₱64.51	5
Transportation, Communication and Storage	0.08%	8	₱39.88	8
Trade	0.12%	7	₱99.31	3
Finance	0.15%	5	₱45.36	7
Real Estate and Ownership of Dwellings	0.02%	10	₱5.91	10
Private Services	0.13%	6	₱88.21	4
Government Services	0.00%	11	₱0.00	11

Economic Loss in Million Php: ₱5,036.22

Print

Static Results

Top 5 Sectors (Inoperability)

Sector	Degree of Inoperability
Agriculture, Fishery and Forestry	5.55%
Mining and Quarrying	1.23%
Electricity, Gas and Water	0.33%
Manufacturing	0.25%
Finance	0.15%

Top 5 Sectors (Economic Loss)

Sector	Economic Loss (in Million PhP)
Agriculture, Fishery and Forestry	₱3,809.71
Manufacturing	₱829.91
Trade	₱99.31
Private Services	₱88.21
Electricity, Gas and Water	₱64.51



[IIM Tool](#)
[DIIM Tool](#)

The damage brought by the disaster greatly affects one or more economic sector (inoperability)

Disaster hits the country

Due to the interdependence of the economic sector, other sectors are also affected (using the IIM)

Quantifies the effect of each sector due to the inoperability of one or more sector.

Rank each sector from the most affected sector to the least affected.

DISASTER RISK MANAGEMENT USING IIM



Dynamic Inoperability Input-Output Model (DIIM)

- Unlike the IIM Tool, the DIIM Tool now allows the users to measure the impact of an initial perturbation in a dynamic setting. This means that users can now observe the effects of a disaster while accounting for the duration of the economy's recovery process and the additional disruptions that may take place after the perturbation is initially experienced.



Step 1: Select Country

DIIM Tool

Select Country

✓

Philippines

Japan

INOPERABILITY

ECONOMIC LOSS



Step 2: Select Year and Dissemination

DIIM Tool

Select Country

Philippines

Select Year & Sector

✓

2000 Philippines 11 Sectors

2000 Philippines 60 Sectors

2006 Philippines 11 Sectors

2006 Philippines 60 Sectors

2006 Philippines 240 Sectors

INOPERABILITY	ECONOMIC LOSS



Step 3: Select Type of Disaster and Recovery Period

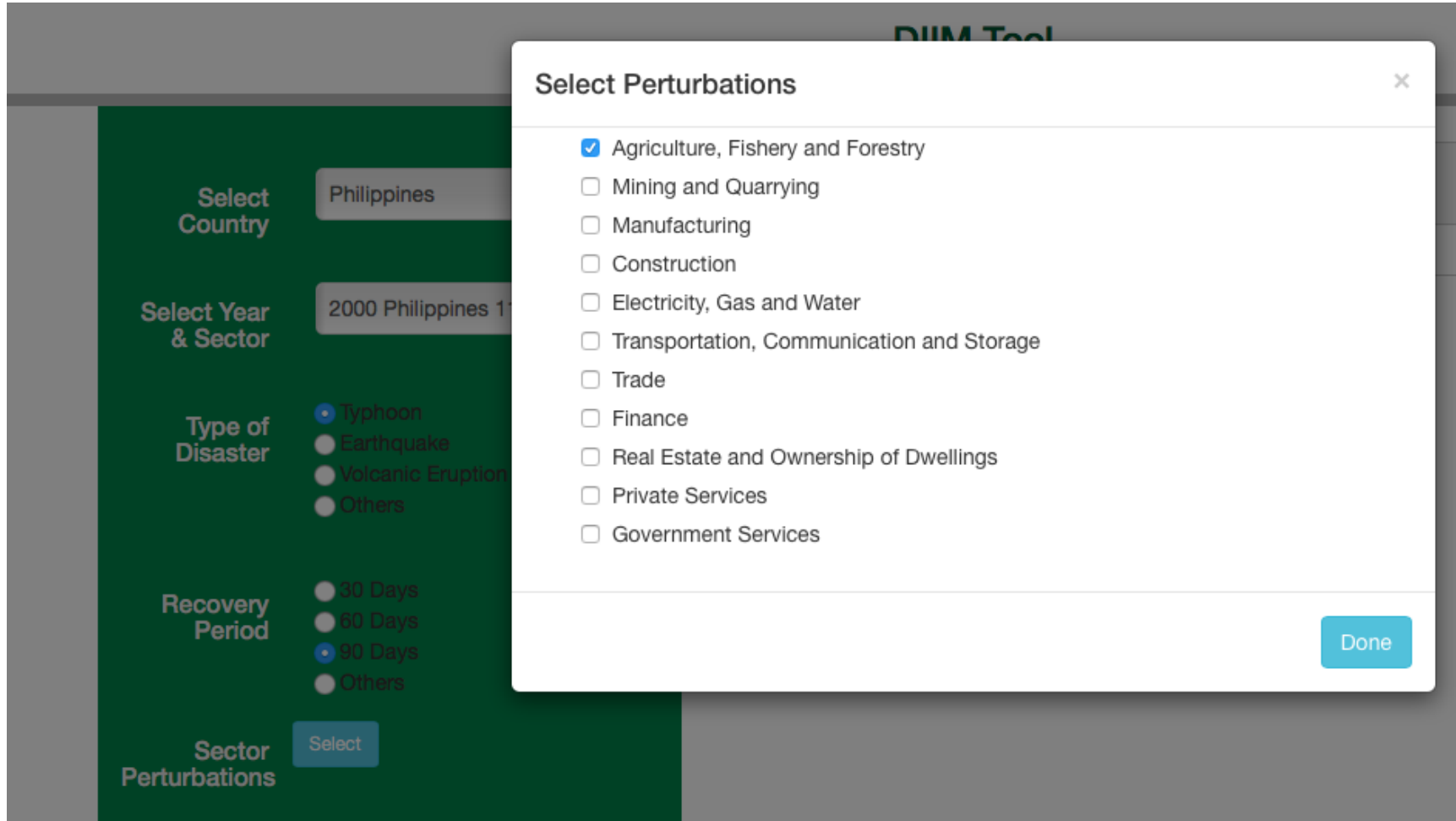
DIIM Tool

Select Country	Philippines
Select Year & Sector	2000 Philippines 11 Sectors
Type of Disaster	<input checked="" type="radio"/> Typhoon <input type="radio"/> Earthquake <input type="radio"/> Volcanic Eruption <input type="radio"/> Others
Recovery Period	<input type="radio"/> 30 Days <input type="radio"/> 60 Days <input checked="" type="radio"/> 90 Days <input type="radio"/> Others

INOPERABILITY	ECONOMIC LOSS



Step 4: Encode Initial Perturbation



The screenshot displays the 'DUM Tool' interface with a modal dialog titled 'Select Perturbations'. The background interface includes the following elements:

- Select Country:** A dropdown menu showing 'Philippines'.
- Select Year & Sector:** A dropdown menu showing '2000 Philippines 1'.
- Type of Disaster:** Radio buttons for 'Typhoon' (selected), 'Earthquake', 'Volcanic Eruption', and 'Others'.
- Recovery Period:** Radio buttons for '30 Days', '60 Days', '90 Days' (selected), and 'Others'.
- Sector Perturbations:** A 'Select' button.

The 'Select Perturbations' dialog box contains a list of sectors with checkboxes:

- ☒ Agriculture, Fishery and Forestry
- ☐ Mining and Quarrying
- ☐ Manufacturing
- ☐ Construction
- ☐ Electricity, Gas and Water
- ☐ Transportation, Communication and Storage
- ☐ Trade
- ☐ Finance
- ☐ Real Estate and Ownership of Dwellings
- ☐ Private Services
- ☐ Government Services

A 'Done' button is located at the bottom right of the dialog box.



Step 4: Encode Initial Perturbation

DIIM Tool

Select Country	Philippines
Select Year & Sector	2000 Philippines 11 Sectors
Type of Disaster	<input checked="" type="checkbox"/> Typhoon <input type="checkbox"/> Earthquake <input type="checkbox"/> Volcanic Eruption <input type="checkbox"/> Others
Recovery Period	<input type="checkbox"/> 30 Days <input type="checkbox"/> 60 Days <input checked="" type="checkbox"/> 90 Days <input type="checkbox"/> Others
Sector Perturbations	Select
<i>Initially affected sector/s (in percentage %)</i>	
Agriculture, Fishery and Forestry	5
Submit	

INOPERABILITY	ECONOMIC LOSS



Select Country

Philippines

Select Year & Sector

2000 Philippines 11 Sectors

Type of Disaster

- ☒ Typhoon
☐ Earthquake
☐ Volcanic Eruption
☐ Others

Recovery Period

- ☐ 30 Days
☐ 60 Days
☒ 90 Days
☐ Others

Initially affected sector/s (in percentage %)

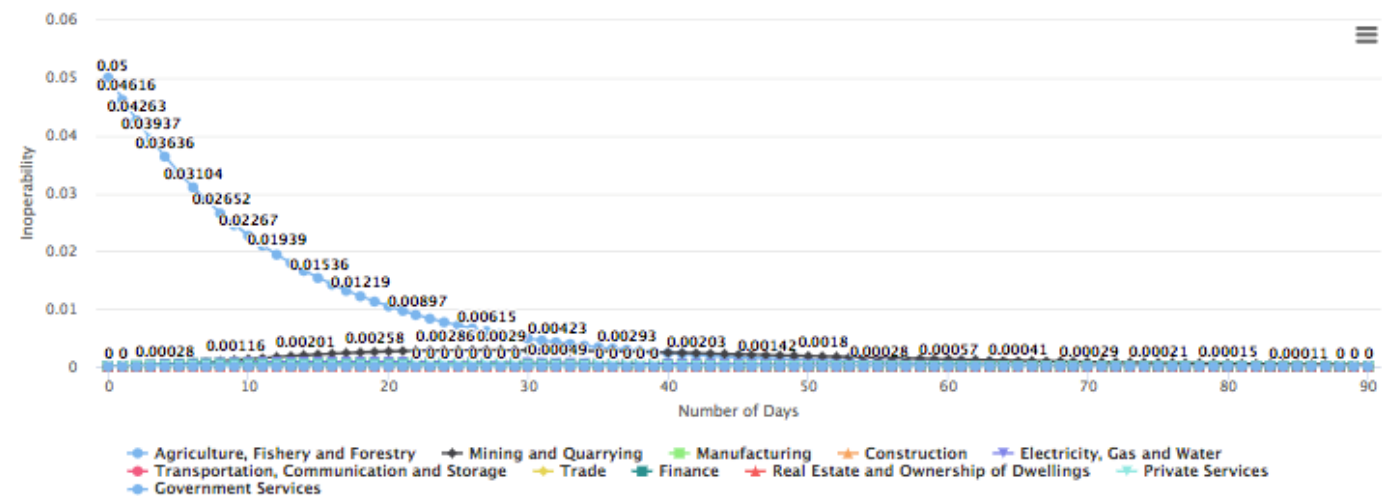
Agriculture, Fishery and Forestry

5

Submit

INOPERABILITY

ECONOMIC LOSS



Highcharts.com

Export to CSV

Sector	Final Inoperability Per Sector	Inoperability Rank
Agriculture, Fishery and Forestry	0.668264	1
Mining and Quarrying	0.141321	2
Manufacturing	0.029808	4
Construction	0.002751	9
Electricity, Gas and Water	0.038441	3
Transportation, Communication and Storage	0.008922	8
Trade	0.014360	7
Finance	0.017713	5
Real Estate and Ownership of Dwellings	0.002330	10
Private Services	0.015698	6
Government Services	0.000000	11

Print

Select Country

Philippines

Select Year & Sector

2000 Philippines 11 Sectors

Type of Disaster

☒ Typhoon

☐ Earthquake

☐ Volcanic Eruption

☐ Others

Recovery Period

☐ 30 Days

☐ 60 Days

☒ 90 Days

☐ Others

Initially affected sector/s (in percentage %)

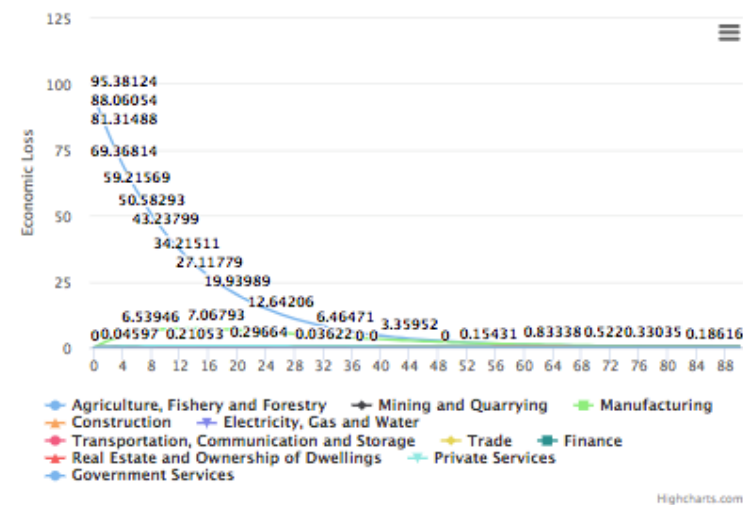
Agriculture, Fishery and Forestry

5

Submit

INOPERABILITY

ECONOMIC LOSS



Export to CSV

Sector	Economic Loss in Million PhP	Economic Loss Rank
Agriculture, Fishery and Forestry	₱1,274.95	1
Mining and Quarrying	₱14.87	6
Manufacturing	₱273.42	2
Construction	₱2.21	9
Electricity, Gas and Water	₱21.03	5
Transportation, Communication and Storage	₱12.76	8
Trade	₱32.11	3
Finance	₱14.80	7
Real Estate and Ownership of Dwellings	₱1.88	10
Private Services	₱28.81	4
Government Services	₱0.00	11

Select Country

Philippines

Select Year & Sector

2000 Philippines 11 Sectors

Type of Disaster

☒ Typhoon
☐ Earthquake
☐ Volcanic Eruption
☐ Others

Recovery Period

☐ 30 Days
☐ 60 Days
☒ 90 Days
☐ Others

Initially affected sector/s (in percentage %)

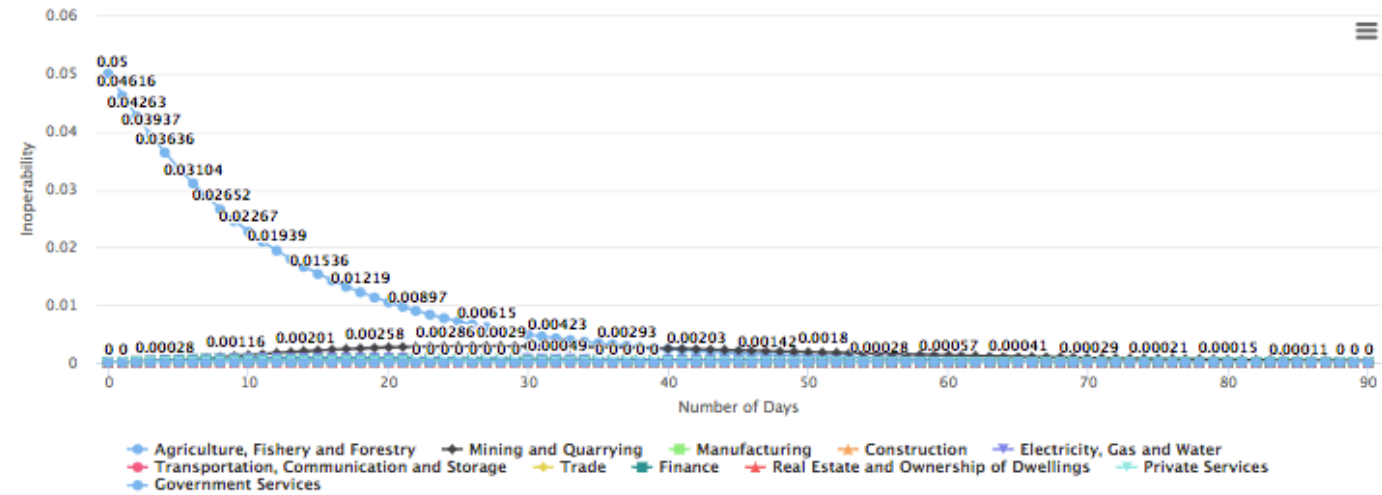
Agriculture, Fishery and Forestry

5

Submit

INOPERABILITY

ECONOMIC LOSS



Highcharts.com

Export to CSV

Sector	Final Inoperability Per Sector	Inoperability Rank
Agriculture, Fishery and Forestry	0.668264	1
Mining and Quarrying	0.141321	2
Manufacturing	0.029808	4
Construction	0.002751	9
Electricity, Gas and Water	0.038441	3
Transportation, Communication and Storage	0.008922	8
Trade	0.014360	7
Finance	0.017713	5
Real Estate and Ownership of Dwellings	0.002330	10
Private Services	0.015698	6
Government Services	0.000000	11

DIIM Tool

Select Country

Select Year & Sector

Type of Disaster

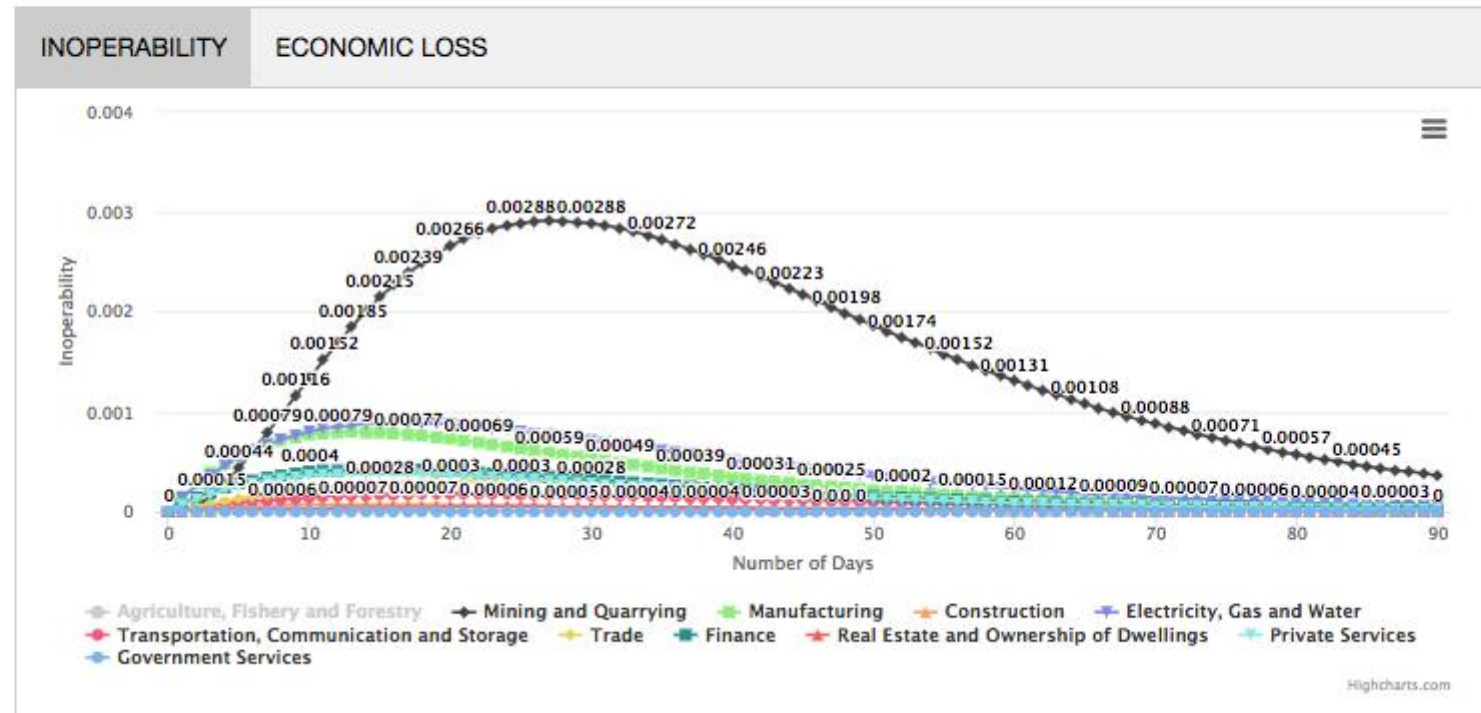
- ☒ Typhoon
- ☐ Earthquake
- ☐ Volcanic Eruption
- ☐ Others

Recovery Period

- ☐ 30 Days
- ☐ 60 Days
- ☐ 90 Days
- ☐ Others

Initially affected sector/s (in percentage %)

Agriculture, Fishery and Forestry



Select Country

Philippines

Select Year & Sector

2000 Philippines 11 Sectors

Type of Disaster

- ☐ Typhoon
☐ Earthquake
☐ Volcanic Eruption
☐ Others

Recovery Period

- ☐ 30 Days
☐ 60 Days
☐ 90 Days
☐ Others

Initially affected sector/s (in percentage %)

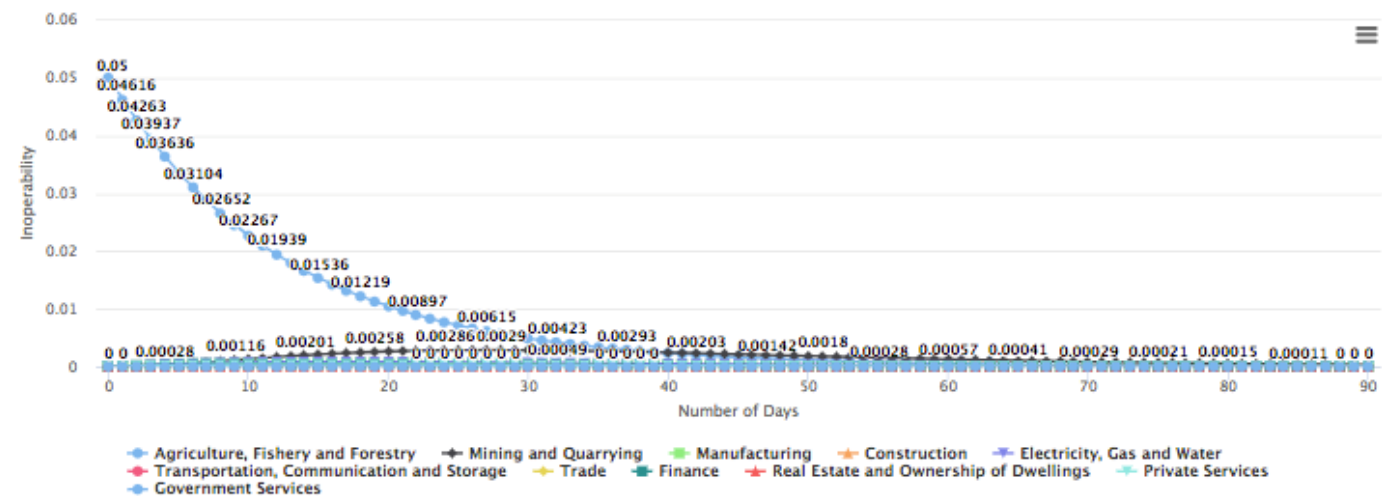
Agriculture, Fishery and Forestry

5

Submit

INOPERABILITY

ECONOMIC LOSS



Highcharts.com

Export to CSV

Sector	Final Inoperability Per Sector	Inoperability Rank
Agriculture, Fishery and Forestry	0.668264	1
Mining and Quarrying	0.141321	2
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Trade	0.014360	7
Finance	0.017713	5
Real Estate and Ownership of Dwellings	0.002330	10
Private Services	0.015698	6
Government Services	0.000000	11

Print

Dynamic Results

Top 5 Sectors (Inoperability)

Sector	Degree of Inoperability
Agriculture, Fishery and Forestry	0.668264
Mining and Quarrying	0.141321
Electricity, Gas and Water	0.038441
Manufacturing	0.029808
Finance	0.017713

Top 5 Sectors (Economic Loss)

Sector	Economic Loss (in Million PhP)
Agriculture, Fishery and Forestry	₱1,274.95
Manufacturing	₱273.42
Trade	₱32.11
Private Services	₱28.81
Electricity, Gas and Water	₱21.03



SUMMARY

- Disaster-REALM's capabilities illustrated through a case study:
 - Overall, results show that some of the most heavily affected industries include trade and finance, whose operations are often thought to be unconstrained by disruptions experienced by the agriculture sector.
 - Results highlight the fact that this powerful tool is able to reveal unapparent connections and interdependencies between sectors within the economy.
- Hence, this stresses the importance of assessing the impact and potential risk of disasters using a tool that is able to account for the complex structure of the economy.



CONCLUSION

- Disaster-REALM
 - web-based freeware that allows users to measure the impact of disasters by simulating an initial perturbation
 - Foundation: Input-Output analysis
 - accounts for the inter-industry relationships within the economy
 - allows users to measure the per-industry effects of a disaster when industries are interdependent
 - Components:
 - IIM Tool: measures the immediate impact of disasters
 - DIIM Tool: measures the dynamic impact of disasters by accounting for the dynamic nature of disruptions and recovery processes of economies



CONCLUSION

- Disaster-REALM
 - aims to help policymakers strategize optimal policy measures by creating summary charts and graphs that report each sector's economic losses and degree of inoperability
 - ranks the sectors according to those that are most at risk in the event of a specific disaster in order to determine the vulnerable sectors that policy measures should be designed towards



RECOMMENDATIONS

- The features of this tool may also be explored by measuring the impact of other hazards such as:
 - drought, earthquakes, pest infestation, reduction in oil production, food supply shortage, pandemics, and workforce absenteeism which may result from other climate change induced disasters



disaster-realm.net



Disaster R.E.A.L.M.

Disaster Estimation and
Analysis with Leontief Models


ECONOMICS SECTOR

Home

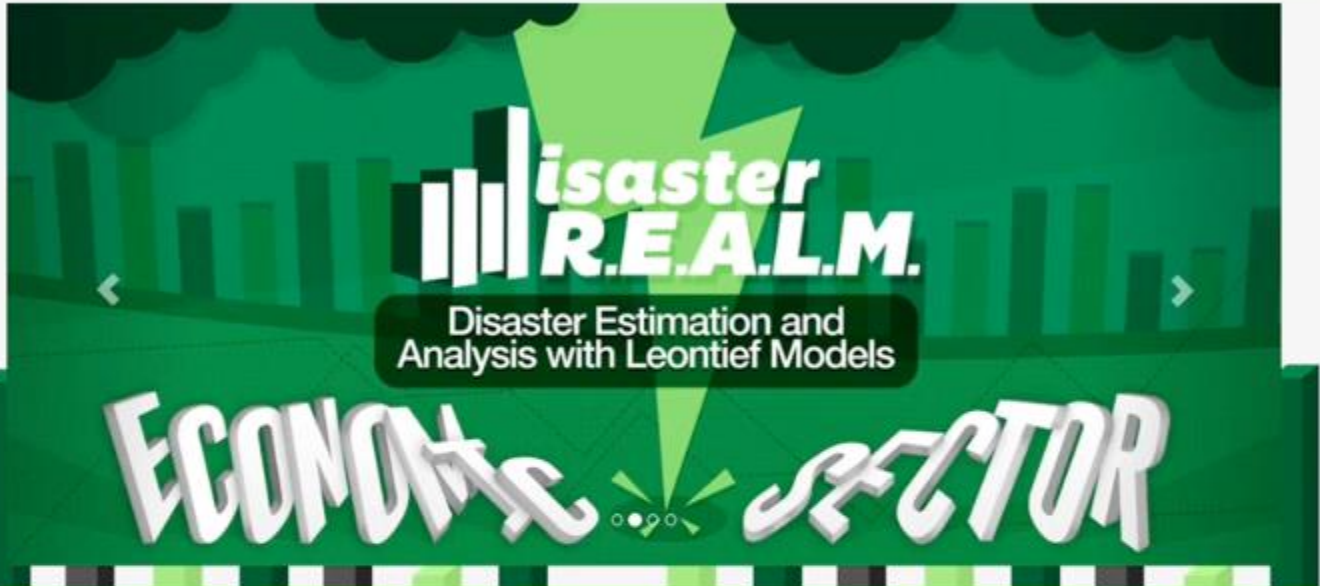
disaster-realm.net/home

☆

⋮

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Language|言語 -



Announcements

- Workshop on Modelling the Economic Impact of Disasters:Static and Dynamic Analysis
- Call for Participants: Special tutorial session on Input-Output Models for DRM and CCA

Read More

Downloadable Files and Links

- 2012 NAST CCC Modeling Philippine Climate Risks Using IIM
- 2013 NAST CCC Developing an MRIO for power shortage mindanao
- Workshop on Modelling the Economic Impact of Disasters:Static and Dynamic Analysis-Files
 - Disasters.pdf
 - An Introduction to Inoperability Input-Output Modeling(IIM) as a Tool for Disaster Risk Management
 - Dynamic Inoperability Input-Output Approach
 - Case Studies
 - Input Output Tables of the Philippines-Philippine Statistics Authority

View More

Partners

- Oscar M. Lopez Center
- The George Washington University



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You may address all comments and suggestions to support@disaster-realm.net
Feedbacks

Notes

Inoperability Input-Output Model

Developed by Haimes and Jiang (2001)

Inoperability is defined as:

- “the inability of a system to perform its intended function” as a result of a perturbation or external shock (Haimes and Jiang, 2001)
- the normalized production loss caused by perturbations



Inoperability Input-Output Model

Inoperability is defined as:

$$\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^* \text{ or } \mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{c}^*$$

where \mathbf{q} = inoperability vector

with each element $q_i = \frac{x_i - \tilde{x}_i}{x_i}$

ranges from 0 (normal
state of a system)
to 1 (total failure)

where x_i = ideal level of output production

\tilde{x}_i = degraded level of output production




Inoperability Input-Output Model

Inoperability is defined as:

$$\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^* \text{ or } \mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{c}^*$$

where \mathbf{q} = inoperability vector

\mathbf{A}^* = interdependency matrix

 represents the additional inoperability sector i contributes to sector j

$$a_{ij}^* = a_{ij} \frac{x_j}{x_i}$$



Inoperability Input-Output Model

Inoperability is defined as:

$$\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^* \text{ or } \mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{c}^*$$

where \mathbf{q} = inoperability vector

\mathbf{A}^* = interdependency matrix

\mathbf{c}^* = perturbation vector

→ represents the reduction of final demand, normalized according to the ideal output level or
 $\mathbf{c}^* = (\hat{\mathbf{x}})^{-1}(\mathbf{c} - \tilde{\mathbf{c}})$



Inoperability Input-Output Model

Inoperability is defined as:

$$\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^* \text{ or } \mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{c}^*$$

where \mathbf{q} = inoperability vector

\mathbf{A}^* = interdependency matrix

\mathbf{c}^* = perturbation vector

column sums of $(\mathbf{I} - \mathbf{A}^*)^{-1}$
= inoperability multiplier

rate of increase in the level of
inoperability caused by an
additional perturbation



Inoperability Input-Output Model

Inoperability is defined as:

$$\mathbf{q} = \mathbf{A}^* \mathbf{q} + \mathbf{c}^* \text{ or } \mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{c}^*$$

Economic Loss: monetary equivalent of the inoperability caused by a perturbation

$$\mathbf{EL} = \mathbf{q}\mathbf{x}$$



Dynamic Inoperability Input-Output Model

The DIIM is defined by:

$$\mathbf{q}(t + 1) = \mathbf{q}(t) + \mathbf{K}[\mathbf{A}^* \mathbf{q}(t) + \mathbf{c}^*(t) - \mathbf{q}(t)]$$

Where \mathbf{q} = inoperability vector

\mathbf{A}^* = interdependency matrix

\mathbf{c}^* = perturbation vector

\mathbf{K} = resilience matrix

t = time indicator

natural log of the initial inoperability over the inoperability at time when there is full recovery

Diagonal matrix of $k_i = \frac{\ln\left[\frac{q_i(0)}{q_i(T_i)}\right]}{T_i} * \frac{1}{(1-a_{ii})}$ or the interdependency index of sector i

