Dynamic Inoperability Input-Output Approach

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Outline of the Presentation

I. Overview
II. Difference between static and dynamic analysis
III. Introduction to DIIM
IV. 2-sector example
V. Hands-on demonstration
VI. Area for future work

Inoperability Input-Output Model

Santos and Haimes (2004)

$$\mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}$$

where:

$\mathbf{A}^*$ = interdependency matrix

$\mathbf{c}$ = forced demand reduction vector

$\mathbf{I}$ = Identity matrix

$\mathbf{q}$ = inoperability vector

a_{ij}^* = Additional inoperability contributed by sector j to sector i

Dynamic Inoperability I-O Model

Lian and Haimes (2006)

$$\mathbf{q(t+1) = q(t) - K[A^* q(t) - q(t)]}$$

where:

$\mathbf{q(t+1)}$	= inoperability vector at time t+1
$\mathbf{q(t)}$	= inoperability vector at time t
$\mathbf{K}$	= resilience matrix
$\mathbf{A}^*$	= interdependency matrix

Dynamic Inoperability I-O Model

Lian and Haimes (2006)

$$\mathbf{q(t+1)} = \mathbf{q(t)} - \mathbf{K}[\mathbf{A}^*\mathbf{q(t)} - \mathbf{q(t)}]$$

where:

$\mathbf{q(t+1)}$	= inoperability vector at time t+1
$\mathbf{q(t)}$	= inoperability vector at time t
$\mathbf{K}$	= resilience matrix
$\mathbf{A}^*$	= interdependency matrix

time scale = per day basis

Dynamic Inoperability I-O Model

Lian and Haimes (2006)

$$\mathbf{q(t+1) = q(t) - K[A^* q(t) - q(t)]}$$

where:

$\mathbf{q(t+1)}$ = inoperability vector at time t+1

$\mathbf{q(t)}$ = inoperability vector at time t

$\mathbf{K}$ = resilience matrix

$\mathbf{A}^*$ = interdependency matrix

diagonal matrix of k_i

$$k_i = \frac{\ln\left[\frac{q_i(0)}{q_i(T_i)}\right]}{T_i} * \frac{1}{(1 - a_{ii})}$$

Recovery-rate parameter

Interdependency Index of *i*

Dynamic Inoperability I-O Model

- More specifically, when sector *i* is affected, and other sectors remain unaffected

$$q_i(0) > 0$$

$$q_j(0) = 0, j \neq i$$

- The recovery trajectory is

$$\dot{q}_i(t) = -k_i\left(1 - a_{ii}^*\right)q_i(0)$$

$$q_i(t) = e^{-k_i\left(1-a_{ii}^*\right)}q_i(0)$$

A 2 – sector example

	Sector 1	Sector 2	Final Demand (c)	Total Output (x)
Sector 1	0.15	0.25	350	1000
Sector 2	0.20	0.05	1700	2000

$$\mathbf{A}^* = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}$$

$$= \begin{bmatrix} 1/1000 & 0 \\ 0 & 1/2000 \end{bmatrix} \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix} \begin{bmatrix} 1000 & 0 \\ 0 & 2000 \end{bmatrix}$$

$$\mathbf{A}^* = \begin{bmatrix} 0.15 & 0.50 \\ 0.10 & 0.05 \end{bmatrix}$$

A 2 – sector example

$$k_i = \frac{\ln\left[\frac{q_i(0)}{q_i(T_i)}\right]}{T_i} * \frac{1}{(1 - a_{ii})}$$

$$\mathbf{A}^* = \begin{bmatrix} 0.15 & 0.50 \\ 0.10 & 0.05 \end{bmatrix}$$

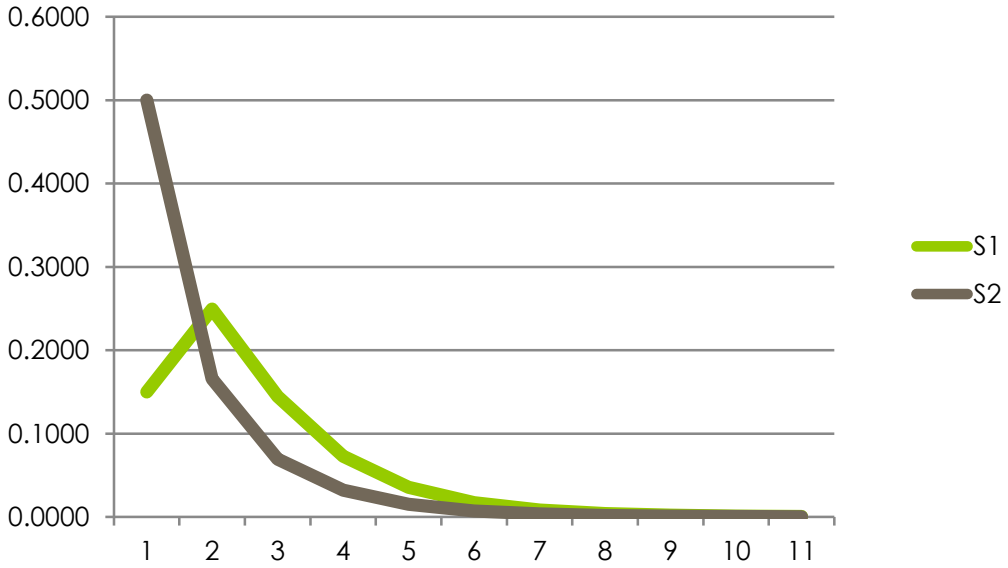
	$\ln\left(\frac{q_i(0)}{q_i(T)}\right)$	T	$\frac{1}{1 - a_{ii}^*}$	k_i
Sector 1	6.9078	10	1.1765	0.8127
Sector 2	9.9078	10	1.0526	0.7271

$$\mathbf{K} = \begin{bmatrix} 0.8127 & 00 \\ 0 & 0.7271 \end{bmatrix}$$

A 2-sector Example

- $\mathbf{q(t+1) = q(t) - K[A^{*}q(t) - q(t)]}$

q0	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10
0.1500	0.2496	0.1444	0.0728	0.0355	0.0171	0.0083	0.0040	0.0019	0.0009	0.0004
0.5000	0.1655	0.0693	0.0319	0.0152	0.0073	0.0035	0.0017	0.0008	0.0004	0.0002



A 2-sector Example

Economic loss of sector *i* = $x_i q_i$

	q0	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10
S1	0.1500	0.2496	0.1444	0.0728	0.0355	0.0171	0.0083	0.0040	0.0019	0.0009	0.0004
S2	0.5000	0.1655	0.0693	0.0319	0.0152	0.0073	0.0035	0.0017	0.0008	0.0004	0.0002

x (Annual)	x(daily)
1000	2.7778
2000	5.5556

	EL(0)	EL(1)	EL(2)	EL(3)	EL(4)	EL(5)	EL(6)	EL(7)	EL(8)	EL(9)	EL(10)		Total Loss
S1	0.4167	0.6932	0.4012	0.2023	0.0986	0.0476	0.0229	0.0110	0.0053	0.0026	0.0012		1.9026
S2	1.3889	0.4598	0.1926	0.0887	0.0421	0.0202	0.0097	0.0047	0.0022	0.0011	0.0005		2.2105
													4.1132

Hands-on Exercise

Areas for Future Work

- Implementation on localized input-output tables (regional or provincial)
- Introduction of recovery strategies
- Successive extreme events

Thank you!

Questions and Comments are Welcome

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